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Board of Trade, Department of Science and Art.

PRACTICAL GEOMETRY.

THE
COURSE OF CONSTRUCTION
OF
PLANE GEOMETRICAL FIGURES,

USED AS A PART OF THE COURSE OF INSTRUCTION IN THE TRAINING
SCHOOL, MARLBOROUGH HOUSE, AND IN THE SCHOOLS OF
ART IN CONNEXION WITH THE DEPARTMENT
OF SCIENCE AND ART.

BY R. BURCHETT,
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LONDON:
CHAPMAN AND HALL, 193 PICCADILLY.

1855.

183. b. 16.

BIBLIOTHECA·BODLEIANA·

LONDON:
Printed by G. BARCLAY, Castle St. Leicester Sq.

INTRODUCTION.

THE collection of Practical Geometry, here presented to the public, is the course which is passed through by the Masters in Training for Schools in connexion with the Department of Science and Art, at the Training School, Marlborough House; the method of teaching adopted, being that of Lecture, with the drawings made on the Black Board by the teacher; the students making notes at the time, and afterwards making careful drawings with the necessary written descriptions.

The superiority of this method of teaching, over that of placing sheets of geometric figures with printed descriptions before a student, and allowing him to *copy* both, need not be enlarged upon here; to extend this method of oral teaching is the principal object sought to be promoted by the publication of the course.

Added to this, a necessity has arisen for placing the entire course, used at the Training School, within the reach of many who cannot attend the training class at

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Marlborough House, in order that they may prepare themselves for examination upon it; the whole course is given, as being necessary to the teacher, although the whole is by no means necessary to all scholars. It has therefore been thought desirable to suggest to the teacher two courses of Practical Geometry, —the first, consisting merely of those figures necessary to all for the practice of Perspective and of drawing generally; such a course as should be taught in all training and other schools, where it is proposed that drawing should form a part of general education; the second, containing, besides the above, those figures which are eminently useful and suggestive in connexion with the various wants of industrial and ornamental Art, and which may be considered adapted to the wants of the central schools.

In both cases, as short a course has been selected as possible; in neither is given *all* that it is *desirable* each class of students should acquire, but both may be regarded rather as nucleii, around which each teacher may group other figures, according to the opportunities and wants of particular classes and localities.

It is hoped also that the present collection may form a useful and ready book of reference to many who, in various branches of trade and manufacture, would be greatly advantaged by a knowledge of Practical Geometry, especially of those portions which apply to the construction and combination of symmetrical figures.

It is proper at once to announce that the work is *descriptive* not *demonstrative;* several years' experience having furnished ample proof, that although it is necessary that the students of our schools should know how to construct Plane Geometric figures, it is vain, as it is perhaps unnecessary, to hope that they will become mathematicians; their want of preliminary training, and the amount of time necessary, rendering such a result an impossibility.

But this should not prevent the teacher from occasionally employing demonstration according to his ability, rather as an incentive to some than as a necessity to all. In all applications of Science to Art, we may be certain that many more will learn *how* to do, than *why* it is done.

As this work has been preceded by an "Introduction to the Study of Plane Geometric Figures," containing all necessary definitions, &c., none are given in the present work, the student being supposed to be fully acquainted with them.

Following the method adopted in the "Introduction," this course of construction of Plane Geometrical Figures is divided into three sections:—

1. The construction of single figures.
2. The combination of figures.
3. The transformation of figures and quantities.

At the *commencement* of each section will be placed such elementary figures as are necessary to the construction of the figures included in it, and no more.

This method has this advantage—the student is not kept working a number of elementary problems of which he cannot discover the use, and which it is most likely he will forget before he has an opportunity of applying them; but, having drawn a few elementary figures, he sees at once their use, and, by their frequent application, impresses them more fixedly in his mind.

As a work of reference, this method has also the advantage of supplying more readily the information sought than an undivided work.

It has been the endeavour to make this course of the construction of Plane Geometrical figures full and complete, without being *limited* or *overloaded* by the requirements of any specialty, to impart to the students of the schools a knowledge useful in every branch of Art, and only so frequently unapplied from being so commonly unknown.

In the constructive problems of the first section it has been desired to place before the student methods that should meet every condition under which the construction of regular plane geometrical figures could be required of him, from knowing well that, to many, a proposition, differing in any degree in its conditions from one previously known, presents insuperable difficulties.

In the construction of Polygons, an effort has been made to complete the series from a five to an eleven-sided polygon, both inscribed and constructed; and one figure is given by which any polygon, from a five to a twenty-sided, may be drawn; whether or not this may admit of frequent practical application, as an exercise for careful geometric drawing it may be useful.

Another class of figures, of great use in Ornamental Art, Ovals, has been introduced—it is hoped not without advantage: one method, by which every variety of ovoid form may be described to given dimensions, admits of numberless applications in practice; and it is hoped that this course may both enrich the memory and stimulate the inventive powers of the students.

In the constructive problems of the second section it has been endeavoured to place before the student means for the extensive combination of regular geometric figures with each other; the infinite applications of these figures to the requirements of Art and Industry must alike excuse their copiousness and their possible deficiency, for whilst to some it may appear that more are given than are often needed, it is quite possible that wants may exist not suppliable by these figures.

Having in the first section given means for the construction of single figures; in the second, for their combination with each other; the third section is devoted to the conversion or transformation of figures into others, equal to them in area, &c.: the elementary problems of this section will relate, therefore, to the measurement of surfaces, &c.

In such a work selection will, of course, supply the bulk of the material; in the present course it is believed, however, that many are new, and these, it is hoped, may be found useful.

The student seeking to learn is strongly advised to draw his figures large; say, for each line one inch long in the plates, to draw one three inches long himself, and, instead of *copying* the description, to write one of his own, and then compare them.

It is also carefully to be borne in mind that the utmost neatness and care is essential to success in drawing Geometrical Figures; without these, errors will constantly arise, confusing the student, and rendering him doubtful of the truth of the method given, but really resulting from his own carelessness and want of precision.

It has been sought to make the descriptions as terse and brief as consistent with explanation, from a belief that a loose colloquial style, while seemingly intended to aid the understanding, tends really to confuse it.

The works of Pasley on Practical Geometry, and of Baron Dupin on the Application of Geometry to the Arts, will furnish most valuable aids to those who may wish to pursue still further the study and application of Practical Geometry.

MARLBOROUGH HOUSE,
January 31*st*, 1855.

PLANE GEOMETRICAL FIGURES.

ALL Right-lined Plane Geometrical Figures are inclosed by lines, either perpendicular and parallel, perpendicular and oblique, parallel and oblique, or oblique only in their relation to each other.

One or more of the following elementary figures (except 15 and 16) are necessary to the construction of every right-lined figure.

ELEMENTARY PROBLEMS TO SECTION I.

FIGS. 1 and 2.

To bisect a line A B, *either straight or curved.*

From A and B, with any radius greater than the half of A B, describe arcs cutting each other in *c* and *d*.

From *c* draw a right line to *d*, and it will *bisect* line A B.

FIG. 3.

To draw a line perpendicular to a given line A B, *at a point* C *in the line.*

From C, with any radius, cut the line A B in c c.

From *c c* describe arcs cutting in *d*. Draw line C *d*, and it will be perpendicular to A B.

Fig. 4.

To draw a line perpendicular to a given line A B, *from a point* C *above or below the line.*

The description and letters of Fig. 3 apply to this figure also.

Fig. 5.

To draw a line perpendicular to a given line A B, *at its extremity.*

From B, with any radius, describe an arc *c*, having its extremity *c* in the line A B.

From *c*, with the *same radius*, cut arc *c* in *d*.

From *d*, with the *same* radius, cut arc *c* in *e*.

From *d* and *e*, with the same or any other radius, describe arcs cutting in *f*.

Draw line *f* B, it will be perpendicular to line A B at its extremity.

Fig. 6.

To draw a line, &c. as the last—a second method.

Take *any point*, lying out of the line A B, and between its extremities, as C.

With the *radius* C A describe arc *c*, having its extremity *c* in the line A B.

From *c*, with the *same radius*, cut arc *c* in *d*.

From *d*, with the same radius, cut arc *c* in *e*; and from *e*, with the same radius, in *f*. Draw line *f* A, and it will be perpendicular to A B at its extremity.

Fig. 7.

The same—a third method.

As in the last figure take any point C.

With C B as radius, describe an arc cutting A B in *c*, and passing through B.

Draw a line from *c*, passing through C, and cutting the arc in *d*.

Draw line *d* B, and it will be perpendicular to A B at its extremity.

Fig. 8.

To draw a line perpendicular to a given line A B, *from a point* C *above or below the line* A B, *quite, or nearly over its extremity.*

Draw any line from C, cutting A B in *c*.

Bisect C *c* in *d*, by Fig. 1.

With *d* C as radius, describe arc C A; its *point of intersection* with line A B will be perpendicular to C.

If the arc pass beyond the extremity of the line

A B, produce it to cut the arc and join C with the point of intersection.

Fig. 9.

To draw a line parallel to a given line A B, *from a given point* C.

From any point on A B as *c*, with the radius *c* C, describe an arc cutting line A B in *d*.

From *d*, with the *same radius*, describe arc *c*.

From *c*, with *d* C as radius, cut arc *c* in *e*, making arc *c e* equal arc *d* C.

Draw line *e* C, it will be parallel to line A B.

Fig. 10.

To draw a line parallel to a given line A B, *at a distance from it, equal to a given line* C D.

From any two points in line A B, with line C D as radius, describe arcs *c* and *d*.

A tangent drawn to both those arcs will be parallel to line A B, at a distance from it equal to line C D.

Fig. 11.

To draw a line from a point C, *making, with line* B C, *an angle equal to a given angle* A.

From A, with any radius, describe arc *a*.

From C, with the *same* radius, describe arc *b c*.

Make arc *b c* equal arc *a*, as in Fig. 9.

Draw line *c* C, and the angle C will be equal to the given angle.

FIG. 12.

To draw a line from a point D, *lying without line* B C, *making an angle with line* B C *equal to a given angle* A.

Draw line D *a* parallel to B C, by Fig. 9.

At the point D make an angle equal to the given angle A.

Produce the side until it cuts line C B in B.

The line D B will make with line C B an angle equal to the given angle A.

FIG. 13.

To divide a line A B *into any number of equal parts.*

Draw line A *a* at any angle to A B, and B *b* at the same angle on the other side of A B, by Fig. 11.

With any radius set off from A and B on the lines A *a* and B *b*, the number of parts, less one, into which it is required to divide A B, as 1, 2, 3, 4, 5.

Join 1—5, 2—4, 3—3, 4—2, 5—1, and A B is divided into six equal parts.

FIG. 14.

The same—another method.

Draw B C at any angle with A B.

Set off on B C the whole number of parts, as 1, 2, 3, 4, 5.

Join 5 A.

Draw lines from 4, 3, 2, and 1, parallel to 5 A, and A B will be divided into five equal parts.

By a similar method a line, greater or less than a given line, may be divided proportionally to it.

FIG. 15.

To divide a line A B *proportionally to the divided line* C D.

Place A B parallel to C D at any distance from it.

Draw lines from the extremities of A B through the extremities of C D, produce them until they cut in E, making a triangle A E B.

Draw lines from E through the divisions 1, 2, 3, 4, of line C D, cutting line A B.

The divisions on A B will have the proportion to the whole divisions on C D that the whole line A B has to the line C D. The lines will be proportionately divided.

FIG. 16.

The same—another method.

Let B C, *the divided line,* make any angle with B A, *the line to be divided,* at B.

Draw line C A, joining the two other extremities of the lines.

Draw lines from 1, 2, 3, 4, 5, parallel to C A, dividing line A B proportionally to line B C.

By either of those methods plans or other drawings may be proportionately enlarged or diminished.

CONSTRUCTIVE PROBLEMS OF SECTION I.

FIG. 17.

To construct an Equilateral Triangle on a given base, A B.

From A and B, with A B as radius, describe arcs cutting in C.

Draw lines A C and C B.

The triangle A C B is equilateral and equiangular.

FIG. 18.

To construct an Equilateral Triangle of a given altitude, A B.

From both extremities of A B draw lines perpendicular to it, by Fig. 5, as C A and D B.

From A, with any radius, describe a semicircle on C A, and with *its* radius cut off arcs *b b*.

Draw lines from A through *b b*, and produce them until they cut the base B D.

Fig. 19.

To construct an Isoskeles Triangle on a given base, A B, *with a given vertical angle* C.

Produce the base A B to *b*.

Make at A an angle *b* A *c*, equal to the given vertical angle C, by Fig 11. From A, with any radius, describe an arc, cutting lines *c* A and A B in *c* and *d*.

Bisect arc *c d* in *e*.

Join A *e* and produce the line.

Make at B an angle equal to angle *d* A *e*.

Produce the side to meet line A *e*, and an isoskeles triangle will be constructed, having a base and vertical angle equal to the given base and vertical angle.

Fig. 20.

To construct any Triangle, its base A B, *and the angles at its base,* A *and* B, *being given.*

Make C D equal A B. Make angle C equal angle A, and angle D equal angle B, by Fig. 11.

Produce their sides until they meet.

Fig. 21.

To construct any Triangle, its three sides A, B, *and* C, *being given.*

From one extremity of the base, as A, describe an arc with the line C as radius.

From the other extremity, with line B as radius, describe an arc, cutting the first arc in D.

From D draw lines to the extremities of line A, and a triangle will be constructed, having sides equal to the sides given.

To construct any triangle, the two shorter sides, B and C, must *together* be more than equal to the longer side A.

Fig. 22.

To construct a Square upon a given base, A B.

Erect a perpendicular at B, by Fig. 5.

Make B C equal A B.

From A and C, with A B as radius, describe arcs, cutting in D.

Join D A and D C to complete the square.

Fig. 23.

To construct a Square, having a given diagonal A B.

Bisect A B in C.

Draw D E perpendicular to A B at C.

Make C D and C E each equal A C.

Join A D, D B, B E, and E A, and a square is constructed, having a diagonal equal to the diagonal given.

Fig. 24.

To construct an Oblong of two given sides, A B and A C.

Draw A C perpendicular to A B at A.

From B, with A C as radius, and from C, with A B as radius, describe arcs, cutting in D.

Join C D and D A.

Fig. 25.

To construct an Oblong, of which the diagonal, A B, *and one side,* A C, *is given.*

Make D E equal A B.

Bisect D E in F.

From F, with F E as radius, describe a semicircle.

From D, with A C as radius, cut the semicircle in G.

Join D G and G E.

Complete the oblong as in Fig. 24.

Fig. 26.

To construct a Rhombus having a base and angle equal to a given base A B, *and given angle* A.

Make line C D equal line A B.

Make angle C equal angle A.

Produce line C E equal to line A B.

From D and E, with line A B as radius, describe arcs, cutting in F.

Join F E and F D to complete the rhombus.

Fig. 27.

To construct a Rhombus having a given diagonal A B, *and a given side* C D.

Make line E F equal line A B.

From E and F, with line C D as radius, describe arcs, cutting in G and H.

Draw lines E G, G F, F H, and E H, to complete the rhombus.

Fig. 28.

To construct a Rhomboid having adjacent sides equal to two given lines A B *and* A C, *and an angle equal to a given angle* A.

Make line D E equal line A B.

Make angle D equal angle A.

Produce line D F equal to line A C.

From F, with line A B as radius, and from E, with line A C as radius, describe arcs, cutting in G.

Join F G and E G.

Fig. 29.

To construct a Rhomboid having its adjacent sides, A B *and* A C, *and its diagonal* D E, *given.*

Make line F G equal the given diagonal D E.

From F, with line A C as radius, and from G, with line A B as radius, describe arcs, cutting in H.

From F, with line A B, and from G, with line A C as radii, describe arcs, cutting in I.

Join G I and G H, H F and F I.

Fig. 30.

To construct a Trapezium equal to a given Trapezium, A, B, C, D.

Make line E F equal line A B.

Make angle E equal angle A.

Make side E G equal side A C.

From G, with the side C D as radius, and from F, with B D as radius, describe arcs, cutting in H.

Join G H and F H.

Fig. 31.

To construct a Trapezium when the length of the diagonal line A B, *and the angles at its extremities* A *and* B, *are given.*

Make line C D equal the length of the diagonal A B.

Make angles at C equal and correspondent to the angles at A, and angles D equal and correspondent to those at B.

Produce their sides until they cut, completing the trapezium.

Fig. 32.

To construct a Trapezium when two of its adjacent sides, A B *and* A C, *the angle* A *formed by them, and the length of the sides making pairs with them,* D E *and* F G, *are given; or if the length of one of them,* D E, *and the length of the diagonal* H I, *is given.*

Make line K L equal line A B.

Make line K M equal line A C.

Make angle K equal angle A.

From L, with line F G as radius, and from M, with line D E as radius, describe arcs, cutting in N.

Join M N and L N.

If one side, D E, and the length of the diagonal H I, is given,

From M, with line D E as radius, and from K, with line H I as radius, describe arcs, cutting in N.

Draw lines M N and L N as before.

Note.—No figures for the construction of Trapezoids are given, as, except that two of their sides are placed parallel to each other, the methods of constructing them under similar conditions would be the same as those for Trapezia.

Fig. 33.

To construct any regular Polygon, the circumscribing circle being given.

At any point of contact C, draw a tangent A B to the given circle.

From C, with any radius, describe a semicircle, cutting the given circle.

Divide the semicircle into as many equal parts as the polygon is required to have sides, as 1, 2, 3, 4, 5, 6.

Draw lines from C through each division, produce them, and the points where they cut the circle will give the place of the angles of the polygon.

Fig. 34.

The same—another method.

Draw a diameter, A B, of the given circle.

Divide A B into as many equal parts as the polygon is required to have sides, by Fig. 13.

From A and B, with the line A B as radius, describe arcs, cutting in C.

Draw a line from C through the *second* division of the diameter and produce it, cutting the circle in D.

B D will be the side of the required polygon.

Set off B D round the circumference of the circle, and points for the angles of the polygon will be obtained.

Note.—Whatever number of sides the polygon may have, the line from C must always be drawn through the *second* division of the diameter.

Fig. 35.

To construct any regular Polygon of which a line A B *is the given side.*

Make B D perpendicular to A B, and equal to it.

From B, with A B as radius, describe arc D A.

Divide arc D A into as many equal parts as there are required sides in the polygon, as 1, 2, 3, 4, 5.

Draw line B 2.

Bisect line A B, and erect a perpendicular at the bisection, cutting B 2 in C.

From C, with C B as radius, describe a circle.

With line A B as radius, cut off arcs on its circumference, and the chords of those arcs will be the sides of the required polygon.

Note.—As in Fig. 34, the line from B is always drawn to the *second* division, from the perpendicular D B of the arc D A, whatever the number of sides in the polygon.

Fig. 36.

The same—another method.

Produce line A B.

From A, with any radius, describe a semicircle.

Divide the semicircle into as many equal parts as are required sides in the polygon, as 1, 2, 3, 4, 5, 6.

Draw lines through every division *except the first:* produce them.

From A, with A B as radius, cut off A 2 in C.

From C, with the same radius, cut A 3 in D.

From B, A 5 in F, from F, A 4 in E.

Join A C, C D, D E, E F, and F B, to complete the polygon.

Fig. 37.

The same—another method.

Bisect A B.

Erect a perpendicular at its bisection.

From A, with A B as radius, describe arc B C, cutting the perpendicular in C.

Divide arc B C into *six* equal parts.

For a *five*-sided polygon,

From C as centre, with C 5 as radius, describe an arc, cutting the perpendicular below C, and the point of their intersection will be the centre required.

A circle described from C, with C A as radius, will contain a *six*-sided polygon.

If the polygon have *more* sides than *six*, set *up* from C on the perpendicular as many parts of the arc A B C as added to six make the number of sides of the required polygon: thus, for a *seven*-sided, set up *one;* for an *eight*-sided, two; and so on; and 7, 8, 9, 10, &c. will be the centres for circles containing polygons having as many sides, each side being equal to the line A B.

Fig. 38.

To describe or inscribe any regular Polygon lying equidistant about or within a similar Polygon, the side of the required Polygon being given.

If the side of the required polygon A B be greater than the given polygon,

Find the centre C of the given polygon, by bisecting two of its adjacent sides, and erecting perpendiculars on the bisections cutting each other.

Draw the radii C D and C E.

Produce the side of the given polygon, making F G equal A B.

From the extremity of the produced side G draw a line parallel to C D, cutting the produced radius C E in E.

From C, with C E as radius, describe a circle.

From E cut off chords equal to A B, and a polygon having its sides equal to A B will be described lying equidistant about the given polygon.

If the side of the required polygon, H I, be less than the side of the given polygon,

Draw the two radii, C D and C E, as before.

Set off on the side of the given polygon the side of the required polygon, as *a b*.

Draw *b c* parallel to radius *a* C, cutting C D in *c*.

From C, with C *c* as radius, describe a circle; it will contain a polygon having sides equal to H I, lying equidistant within the given polygon.

Fig. 39.

To construct any regular Pentagon, the circumscribing circle A being given.

Draw a diameter A B, and a radius C D perpendicular to it.

Bisect A C in *a*.

From *a*, with *a* D as radius, describe arc *b* D, cutting the diameter in *b*.

Draw line D *b*, it will be equal to the side of the pentagon.

Fig. 40.

To construct a regular Hexagon, the circumscribing circle being given.

Draw a diameter, A B.

From its extremities with the radius of the circle cut off arcs *a b*, *c d*.

Draw lines A *a*, A *b*, *c* B, B *d*, *a c*, and *b d*.

Fig. 41.

To construct a regular Heptagon, the circumscribing circle being given.

Draw a radius, A B.

From B, with A B as radius, describe arcs, cutting the circumference in *a b*.

Draw line *a b*, it will be bisected by the radius in C.

C *a* will be equal to the side of the required heptagon.

FIG. 42.

To construct a regular Octagon, the circumscribing circle being given.

Draw a diameter, A B.

Bisect the arcs A B in C and D.

Bisect arcs C A and C B, in *a* and *b*.

Draw lines from *a* and *b* through the centre of the circle, cutting the circumference in *c* and *d*.

Draw lines A *a*, *a* C, C *b*, *b* B, B *c*, *c* D, D *d*, and *d* A.

FIG. 43.

To construct a regular Nonagon, the circumscribing circle being given.

Draw a produced diameter, A B; another perpendicular to it, C D.

From C, with the radius of the circle, describe an arc cutting arc C B in *a*.

From D, with D *a* as radius, describe an arc, cutting the produced diameter in *b*.

From *b*, with *b* C as radius, describe an arc, cutting A B in *c*.

Line *c* A will be equal to the side of the nonagon.

FIG. 44.

To construct a regular Decagon, or Pentadecagon, the circumscribing circle being given.

Draw a diameter, A B, and a radius perpendicular to it, C D.

Bisect C D in E.

Draw line E A.

From E, with E C as radius, describe an arc, cutting E A in F.

From A, with radius A F, describe an arc, cutting the circumference in G.

The chord A G is the side of the decagon.

From A, with radius A C, describe an arc, cutting the circumference in H.

The chord G H is the side of a pentadecagon.

Fig. 45.

To construct a regular Undecagon, the circumscribing circle being given.

Draw two diameters, A B and D E, perpendicular to each other, intersecting in C.

From E, with radius E C, describe an arc, cutting the arc E A in *a*.

From A, with the same radius, describe an arc, cutting the arcs A D in *b*.

From *a*, with radius *a b*, describe an arc, cutting the radius C D in *c*.

Draw the right line *c b*, it will be equal to the side of the undecagon.

Fig. 46.

To construct a Pentagon upon a given line A B.

Produce A B.

From B, with radius B A, describe a semicircle, cutting the produced line A B in C.

From A, with radius A B, describe an arc, cutting the semicircle in D.

Bisect A B in E.

Draw line D E.

Bisect arc B D in F.

Draw line E F.

From C, with radius E F, cut off on the semicircle arc 1, and from 1 with the same radius arc 2.

Draw line B 2, it will be a second side of the pentagon.

Bisect it, and draw a line perpendicular to it at the bisection.

The perpendiculars from the sides A B and B 2 will cut in G.

From G, with the line G A as radius, describe a circle; it will contain the required pentagon.

Fig. 47.

To construct a Hexagon upon a given line A B.

From A and B, with radius A B, describe arcs cutting in C.

From C, with radius C A, describe a circle; it will contain the required hexagon.

Fig. 48.

To construct a Heptagon upon a given line A B.

Produce A B.

From B, with radius A B, describe a semicircle, cutting the produced line A B in C.

From A, with the same radius, describe an arc, cutting the semicircle in D.

Bisect A B in E.

Draw line D E.

From C, with line D E as radius, cut off on the semicircle arc 1.

Draw line B 1, it is a second side of the heptagon.

Bisect it, and obtain the centre of the circumscribing circle, as in Fig. 46.

Fig. 49.

To construct an Octagon on a given line A B.

Produce line A B.

From B, with radius B A, describe a semicircle, cutting the produced line A B in C.

Bisect the semicircle in D.

Bisect arc C D in 1.

Draw line B 1, it will be a second side of the octagon.

Obtain the centre as before.

Fig. 50.

To construct a Nonagon on a given line A B.

Produce line A B.

From B, with radius B A, describe an arc, cutting

the produced line A B in C, and being produced below A.

From A, with the same radius, describe an arc, cutting the first in D and E.

Draw line D E, cutting A B in F.

From D, with radius D A, describe arc A B.

From E, with radius E F, describe an arc, cutting arc A B in G and H.

From C, with line G H as radius, cut the semicircle in 1.

Draw line B 1, it is a second side of the nonagon.

Find the centre as before.

Fig. 51.

To construct a Decagon on a given line A B.

Produce A B.

From B, with radius B A, describe a semicircle, cutting in C.

From A, with radius A B, describe an arc, cutting the semicircle in D, and bisect A B in E, all as before.

From B, with radius B E, describe an arc, cutting arc B D in F.

Draw line E F.

From C, with line E F as radius, cut the semicircle in 1.

B 1 will be a second side of the decagon.

Obtain the centre of the circumscribing circle as before.

FIG. 52.

To construct an Undecagon on a given line A B.

Produce A B, and describe the semicircle as before, producing it below A.

From A, with radius A B, describe an arc, cutting the first arc in D and E.

Draw the line D E, bisecting A B.

From B, with the half of B A as radius, describe an arc cutting arc B D in G.

Bisect arc B E in H.

Draw lines A G and A H, cutting E D in I and K.

From C, with radius I K, cut off on the semicircle arc 1.

Draw line B 1, and proceed as before.

FIG. 53.

Within a given circle to find the side of any Polygon, from a five to a twenty-sided, inclusive.

Draw a diameter, A B.

Draw a radius, C D, perpendicular to it.

Bisect A C in E.

Join E D.

From E, with radius E D, describe an arc, cutting C B in F.

The chord D F *is the side of a pentagon.*

The radius C D *is the side of a hexagon.*

From C, with radius C F, describe an arc, cutting C D in G.

Join G F.

Bisect arc F D in H.

Join E H, cutting C D in I, and produce the line to cut the circumference in K.

At angle C upon C F construct a square, having sides equal to C I; it will touch line F G in L, bisecting C F in M.

Join L E, *and the line is the side of a heptagon.*

Bisect angle L E D, and produce the bisecting line, cutting C D in N.

Join N F, *and the line is the side of an octagon.*

Join F I, cutting L M in O.

Join K O, *and the line is the side of a nonagon.*

Line C F *is the side of a decagon.*

Draw line L K, *it is the side of an undecagon.*

From E, with radius E C, describe an arc, cutting line E N in P.

Draw line L B, cutting O K in Q.

Join P Q, *it is the side of a dodecagon.*

Line O K cuts arc F G in R.

Draw I R, *it is the side of a tridecagon.*

Line L K cuts D F in S.

Join S G, *it is the side of a quadecagon.*

Line E K cuts line F G in T.

From L, with L T as radius, describe an arc, cutting the top of the square L I in U.

Join U H, *the line is the side of a pentadecagon.*

Line P Q cuts line E L in V.

Join V C, *it is the side of a hexadecagon.*

Line L H cuts line G S in W.

Join I W, *it is the side of a heptadecagon.*

Line I O *is the side of an octadecagon.*

Line U H cuts line I W in X.

Join P X, *it is the side of a nonadecagon.*

Line L H *is the side of a bisdecagon.*

Fig. 54.

To describe an Ellipse, its axes or transverse and conjugate diameters A B *and* C D *being given.*

Place the transverse diameter A B and the conjugate diameter C D perpendicular to each other at their centres.

From C or D, with the half of A B as radius, describe arcs, cutting line A B in F 1 and F 2.

These are the focii of the ellipse.

Divide the two portions of the line A B lying between the two focii and the centre of line A B into any number of parts; they may be equal, but it will be an advantage if the divisions lessen as they approach F.

Number them 1, 2, 3, 4, &c.

From F 1, with radius A 1, A 2, A 3, A 4, &c. describe arcs in the spaces A C and A D.

From F 2, with B 1 (the first division towards A beyond the centre of A B), B 2, B 3, B 4, &c. describe arcs cutting the arcs already described from F 1; radius B 1 cutting arc A 1, &c. in *a, b, c, d,* &c.

Draw a line, passing through the points A *a, b, c, d, e, f,* C, and complete the figure by a repetition of the process.

Fig. 55.

The same—another method.

Place the diameters perpendicular to each other at their centres C, as before.

From C, with radii C A and C D, describe circles.

Divide the circumference of the larger circle into any number of equal parts, 1, 2, 3, 4, 5, 6, 7.

Draw radii from each division, cutting the smaller circle also in 1, 2, 3, 5, 6, 7.

From the divisions of the smaller circle, draw lines parallel to the transverse diameter A B.

From the divisions of the greater circle, draw lines parallel to the conjugate diameter D E.

The intersections of the lines so drawn will give points through which to draw the curve of the ellipse.

Fig. 56.

The same—another method.

Place the diameters as before.

At A and B draw lines parallel, and equal, to line D E.

At D and E draw lines parallel, and equal, to line A B, constructing an oblong, F, G, H, I, having sides equal to the diameters of the ellipse.

Divide line A C and B C into the same number of equal parts, 1, 2, 3, C.

Divide lines A F and B G into the same number of equal parts, as lines A C and B C.

Draw lines from E through the divisions on line A B.

Draw lines to D from the divisions on lines A F and B G.

The intersections will give points as in the last figure.

Repeat the process for the other half.

Fig. 57.

The same—another method.

Place the diameters—find the focii and letter them, as in Fig. 54.

Fix two points, as pins, at F 1 and F 2.

Tie loops at the ends of a string, the string, including the loops, when pulled tight, being equal in length to line F 1, D, F 2.

Place the loops over the pins at F 1 and F 2.

Place a pencil or other marking point within and against the string (which must be kept equally strained).

Make the pencil traverse from A through D to B, and if care has been taken to keep the string equally strained, a semi-ellipse will be obtained.

Turn the string over and repeat the process for the other half.

Fig. 58.

To describe an Elliptical figure by Arcs of Circles, its transverse and conjugate diameters A B *and* D E *being given.*

Place the diameters perpendicular to each other at their centres, C.

From A, with line D E as radius, cut line A B in F.

Divide F B into three equal parts.

From C, with two of those parts as radii, cut line A B in *a* and *b*.

From *a* and *b*, with radius *a b*, describe arcs cutting in *c* and *d*.

From *d*, with radius *d* D, describe arc *e f*.

From *a* and *b*, with radius *a* A, describe arcs *e g* and *f h*.

From *c*, with radius *c* E, describe arc *g h* to complete the figure.

Lines drawn from *c* and *d*, through *a* and *b*, will give the points of junction of the four arcs.

Note.—*This figure is given as a simple mechanical method of producing a figure approximating to an ellipse, but it is on many accounts objectionable from the imper-*

fections of the curve obtained, and the frequent impossibility of the coincidence of the arcs described from the two sets of centres.

Fig. 59.

To describe an Oval or Egg-shaped figure, its transverse and conjugate diameters A B *and* C D, *and their point of intersection* E, *being given, by points and a string.*

Place the diameters perpendicular to each other at E, making E C equal E D.

Bisect A E and draw a line through the bisection parallel to C D.

From A, with radius E C, cut the last drawn line in G and H.

From A, with the half of line G H as radius, cut line G H in I and K.

Divide G I into *two* equal parts, and set off *one* of them from I and K, cutting line G H in F 1 and F 2.

From B set off *two* of the same parts, cutting line A B in F 3.

Place pins in A, F 1, F 2, and F 3.

Tie a string tightly round those four pins.

Remove the pin at A and substitute a marking point, as in Fig. 57.

The traversing point will describe an oval passing through, and having its greatest dimensions in, the points A, B, C, D.

Fig. 60.

To describe an Oval by Arcs of Circles.

Upon any right line, A B, describe a semicircle C D, equal in diameter to the proposed oval.

From C and D, with the radius of the semicircle, cut the right line in A and B.

From A and B, with radius B C, describe arcs C E and D F.

From A or B draw a right line through the transverse diameter, cutting it in G, touching the opposite arc in E or F.

From G, with radius G E, describe arc E F, completing the figure.

Note.—*The length of the figure may be increased or diminished by increasing or diminishing the radii* A D *and* B C, *and by placing the point* G *higher or lower on the transverse diameter.*

Fig. 61.

To describe a Spiral Curve, the greatest diameter A B *being given.*

Bisect line A B in C.

Divide B C into *four* equal parts.

Upon the *first* part *below* C describe a circle equal in diameter to the part, this will be the eye of the spiral.

Then, as in the larger figure D,

Inscribe a square, and draw its central diameters, 1—3, 2—4.

Divide each diameter into *six* equal parts.

Number them as in the figure—their extremities, commencing at the top left, being 1, 2, 3, 4—the first divisions on each 5, 6, 7, 8—the second 9, 10, 11, 12.

From 1, with radius 1 A, describe an arc.

From 1 draw a line through 2, and produce it to cut the arc last drawn in *a*.

From 2, with radius 2 *a*, describe an arc, terminating it by a line drawn from 2 through 3, and proceed thus, describing arcs from each point in the eye until the spiral is complete, always terminating the arcs, as in arc 2, by drawing a line from their centres *through the point next in advance*.

ELEMENTARY PROBLEMS OF SECTION II.

Fig. 62.

To bisect a given Angle A.

From the angular point A, with any radius, describe an arc cutting the sides of the angle in *a* and *b*.

From *a* and *b* describe arcs cutting in *c*.

Draw line A *c*, and the angle is bisected.

Fig. 63.

To trisect a Right Angle A.

From the angular point A, with any radius, describe an arc cutting the sides of the angle in *a* and *b*.

From *a* and *b*, with the same radius, cut off the arcs *c* and *d*.

Draw lines *c* A and *d* A and the *right* angle will be trisected.

FIG. 64.

To find the centre of any Triangle A B C, *viz. a point equidistant from all its sides.*

Bisect any two of its angles, as A C, by Fig. 62.

Draw the bisecting lines until they cut.

The point of intersection, D, will be the centre.

FIG. 65.

To find the Centre of any Circle A.

Draw any chord, B C.

Bisect it in D.

Draw a line through D perpendicular to B C, its extremities touching the circumference; it is a diameter.

Bisect it, and the centre is found.

FIG. 66.

To find the Centre and Axes or Transverse and Conjugate Diameters of an Ellipse, the circumference A *being given.*

Draw any two parallel chords, A B and D E.

Bisect them in *a* and *b*.

Draw a line F G through *a b*, its extremities touching the circumference; it is a diameter.

Bisect the diameter in C.

C is the centre.

From C, with any radius, describe arcs cutting the circumference in *c*, *d*, and *e*.

Draw lines *c d* and *d e*.

Draw, through C, lines H and I, parallel to lines *c d* and *d e*.

H and I will be the axes, or transverse and conjugate diameters sought.

Fig. 67.

To describe a Circle or an Arc passing through any three given points, A, B, C, *such points not lying in a straight line.*

Join points A B and C by right lines.

Bisect lines A B and B C.

Erect perpendiculars at their bisections; they will intersect each other in D.

D will be the centre, and D A the radius of an arc or circle passing through the given points.

Fig. 68.

To draw a Tangent to a Circle at a given point of contact A.

Draw a radius A H to the given point of contact.

At A draw a line C D perpendicular to A H by Fig. 3.

C D will be the tangent required.

FIG. 69.

To draw a Tangent to a Circle from a given point A *lying without the circumference.*

Draw a line from the given point A to the centre of the circle C.

Bisect line A C in B.

From B, with radius B A, describe a semicircle, cutting the given circle in D.

Draw line A D and produce it; it is the required tangent drawn from the given point.

FIG. 70.

To draw a Tangent to any point of contact A, *in the arc of a circle* A B, *when the centre cannot be obtained or used.*

Draw the *chord,* A B.

Bisect it in C.

Erect C D perpendicular to A B.

Draw line D A.

Make angle D A E equal angle D A C.

Produce the line E A; it is the tangent required.

FIG. 71.

To draw a Tangent to an Ellipse at a given point of contact A.

Draw the transverse axis by Fig. 66, and obtain the focii *a b* of the ellipse by Fig. 57.

From *a* and *b* draw lines *a* A and *b* A through the given point of contact, producing one of them, as *a* A to *c*.

Bisect the external angle *c* A. *b* in B.

Draw line B A and produce it; it is the tangent required drawn through the given point.

Fig. 72.

To draw a Line perpendicular to the curve of an Ellipse from a given point A.

Draw the transverse axis, and obtain the focii as in the last figure.

Draw lines from the focii through the given point A, producing them both, making angle *a* A *b*.

Bisect angle *a* A *b* by line B A; it will be perpendicular to the curve.

By this process the joints of masonry for elliptic arches may be obtained.

CONSTRUCTIVE PROBLEMS OF SECTION II.

Fig. 73.

To construct a Triangle having a Perimeter (or measure about) equal to a given line A B, *and angles equal to those of a given triangle* C.

On the given line A B construct a triangle A B D, having angles equal to those of the given triangle C.

Find its centre E by Fig. 65.

From E draw lines parallel to the sides of the triangle B D and A D, cutting the line A B in F and G

Triangle E F G will have a perimeter equal to the given line A B, and angles equal to those of the given triangle C.

Fig. 74.

To inscribe or describe any similar Triangle, having a given side A B *or* F G, *within or about and equidistant from the sides of the given triangle* C D E.

Bisect the angles of the triangle by lines C F, D F, and E F.

Set off on line C D the given side A B as C *a*.

Draw a line *a b* parallel to the line C F, cutting the bisecting line D F in *b*.

From *b* draw lines parallel to the sides of the triangle, intersecting the bisecting lines E F and D F in *c* and *e*.

If the triangle be required to be described about the given triangle having a side equal to the line F G,

Produce D E to G, making D G equal to F G.

Bisect the angles as before, and produce their bisecting lines without the triangle.

From G draw a line parallel to the bisecting line D F, cutting the produced line F E in *f*.

From *f* draw lines parallel to the sides of the triangle as before.

The side of the triangle *f g* will, in this case, be equal to the line F G: if the side parallel to C E were required to be equal to the given line F G, then the line C E must have been produced equal to line F G, and from its extremity a line drawn parallel to C F, and so of the other side.

FIG. 75.

To inscribe a Square within any Triangle, A B C.

From the vertical angle B draw a line B D, perpendicular to the base A C; this is the altitude or vertical height of the triangle.

From B draw a line E, parallel to the base A C, and equal to the line B D, the altitude.

Draw from the angle opposite to E, as angle C, a line C E, cutting the side of the triangle A B in F.

From F draw a line F G, perpendicular to the base A B.

Draw line F H parallel to A B, cutting the side B C in H.

Draw line H I parallel to F G, and the square will be complete.

Note.—*The side of the square may be coincident with any side of the triangle, if neither of the angles at the extremity of such side is greater than a right angle.*

Fig. 76.

To inscribe an Oblong in any Triangle, as D E F, *such oblong having a side equal to a given line* A B, *not greater than the longest side of the triangle.*

Set off D C on the base of the triangle equal to line A B.

From C draw a line parallel to side D E, cutting side E F in K.

From K draw a line perpendicular to the base, and another parallel to the base, cutting sides D E and D F in H and I.

From H draw a line perpendicular to the base, to complete the oblong.

Fig. 77.

To inscribe a Circle within any Triangle.

Find the centre, C, by Fig. 65.

From C draw a line C D, perpendicular to any side of the triangle; it will be the radius of the circle to be inscribed.

Fig. 78.

To describe a Circle about any Triangle.

Bisect any two sides of the triangle.

From the points of bisection A B draw lines perpendicular to those sides, produce them until they cut.

Their point of intersection, C, will be the centre, and a line drawn from such point to any angle of the triangle, as C D, the radius of the circle.

This figure is, in fact, only another application of Fig. 68, the three angles of the triangle being similar to the three points there given.

Fig. 79.

To inscribe an Equilateral Triangle in a Square.

Trisect angle A B C in *a* and *b* by Fig. 63.

Bisect angles A B *a* and *b* B C in D and E.

Draw lines D B, D E, and E B, to complete the triangle.

Fig. 80.

To inscribe an Isoskeles Triangle in a Square, such triangle to have a base equal to a given line A B, *not greater than the diagonal of the square.*

Draw a diagonal, C D.

From C, with *half* line A B as radius, cut C D in G.

Draw a line through G, perpendicular to C D, cutting the sides of the square in E and F.

Draw lines E C and C F to complete the triangle.

Fig. 81.

To describe an Equilateral Triangle about a Square, A B C D.

From C and D, with the side of the square as radius, describe arcs A E and B F, cutting each other in G.

From G, with the same radius, cut arcs A E and B F in E and F.

Draw lines E D and F C, and produce them to H.

Produce the base A B until it cuts lines D E and C F in I and K.

H I K will be an equilateral triangle.

FIG. 82.

To inscribe or describe a Square, having a side equal to a given line A B, *within or about, and equidistant from, the sides of a given square.*

This figure is another application of Fig. 75.

Draw the diagonals C D and E F.

Make F G equal line A B.

From G draw a line G H, parallel to F E.

Draw a line from H parallel to D F, and complete the figure as in 75.

If the side be greater than the given side, proceed as in Fig. 75.

FIG. 83.

To inscribe or describe a Circle within or about a given Square.

Draw the two diagonals of the square, intersecting each other in C.

From C draw a line perpendicular to any side, as C A.

C will be the centre, and line C A the radius of the circle to be *inscribed.*

C will be the centre, and half the diagonal, as C B, the radius of the circle to be *described* about the given square.

Fig. 84.

To inscribe a Square in any Equilateral Four-sided Figure.

Draw the two diagonals of the given figure A B, D E, intersecting at C.

Bisect any two adjacent angles, as A C E and B C E.

Produce their bisections, cutting the sides of the given figure in F, G, H, I.

Draw lines F, G, H, I, to complete the figure.

Fig. 85.

To inscribe a Rhombus in any Parallelogram, whether right or oblique-angled.

Draw the diagonals and bisect the angles as before, but if it is required that the angle of the inscribed figure should touch any particular point of the given figure, as G, then (instead of bisecting the angles) from G draw a right line through the centre C, and another perpendicular to it, also through C; the extremities of those lines will give the points F, G, H, I.

Fig. 86.

To inscribe a Square in any Trapezium having adjacent pairs of sides equal.

Draw the diagonals of the trapezium A B and C D.

Divide either of the sides of the trapezium, as A C,

proportionally to the two diagonals, by Fig. 15; that is, draw a line equal to A B, as E F.

Produce it, making F G equal to C D.

Place H I equal to A C parallel to E G.

Construct the triangle E K G.

Draw a line from F to K, cutting line H I in L.

From C, with radius I L, cut line A C in M.

Draw M N parallel to A B, and upon the line M N construct the required square.

Fig. 87.

To inscribe a Circle in any Equilateral Four-sided Figure, A B D E.

Bisect any two adjacent angles, as A and B; their bisections will cut in C.

From C draw a line perpendicular to any side of the given figure, as C F.

C will be the centre, and C F the radius, of the circle to be inscribed.

Fig. 88.

To inscribe a Circle in any Trapezium, A B D E, *having adjacent sides equal.*

The description and letters of the last figure apply also to this.

Fig. 89.

To inscribe or describe an Equilateral Triangle within or about a given Circle.

Draw a diameter of the circle, A B.

From A, with the radius of the circle, cut its circumference in C and D.

Draw lines C B, C D, and B D, for the *inscribed* triangle.

From B, C, and D, with line B C as radius, describe arcs cutting in E, F, and G.

Draw lines E F, E G, and F G, for the *described* triangle.

Fig. 90.

To inscribe any Triangle in a Circle, such triangle to have angles equal to those of a given triangle A B C.

Draw a tangent, D E, to the circle.

At its point of contact, F, make angle D F G equal angle A.

Make angle E F H equal angle C.

The sides of the angles F G and F H cutting the circumference in G and H.

Draw line G H, and F G H will be the inscribed triangle, having its angles equal to those of the given triangle.

Fig. 91.

To describe any Triangle about a Circle, such triangle to have angles equal to those of a given triangle A B C.

Produce the base of the given triangle, as *a b*.

Draw any radius, D E, of the circle, and another D F, making angle E D F equal to the external angle of the triangle *a* A C.

Draw another radius, D G, making angle E D G equal to the external angle *b* B C.

At the extremities of the radii E F G draw tangents, cutting in H I K.

The triangle H I K will be described about the given circle, and its angles will be equal to those of the given triangle.

Fig. 92.

To inscribe or describe a Square in or about a Circle.

Draw two diameters, A B and C D, perpendicular to each other.

Join their extremities by lines A C, C B, B D, and A D, for the *inscribed* square.

From A, B, C, and D, with the radius of the circle, describe arcs cutting in E, F, G, and H.

Draw lines E F, E G, F H, and G H, for the *described* square.

Fig. 93.

To describe a Circle touching two given Circles A *and* B, *and one of them in a given point* C.

Draw a right line, A B, connecting the centres of the given circles.

Draw from C, the proposed point of contact, a radius, C B.

In the second circle draw a radius, A D, parallel to radius C B.

Draw line C D, producing it if necessary to a point opposite to C, as E.

Draw lines C B and E A, producing them until they cut in F.

F C will be the radius of the circumscribing circle.

Note.—This problem will apply in all cases, whether the circles be equal or unequal, in contact, intersecting, inscribed, or separated, except when the line C D is a tangent to both circles.

Fig. 94.

To divide a Circle into any given number of proportional parts by concentric divisions.

When the parts are required to have a progressional proportion, as 1, 2, 3, 4, &c., to each other, draw a radius, A B, of the given circle, and divide it into as

many equal parts as are required divisions in the circle, as 4.

Upon the radius A B, with its half as radius, describe a semicircle, A C B.

Erect perpendiculars upon the radius at the subdivisions 1, 2, 3, 4, cutting the semicircle, A C B, in 5 C 6.

From A as centre, and with A 6, A C, and A 5, as radii, describe circles.

The circle A 6 will be one-fourth; A C one-half; A 5 three-fourths of the original circle.

Note.—The number of parts admit of any extension, and, if required, may be unequal.

Fig. 95.

To divide a Circle into any number of parts equal to each other in area and perimeter.

Draw a diameter, A B.

Divide it into as many equal parts as are required divisions of the circle, as 1, 2, 3.

Upon A 1, with its half as radius, describe a semicircle.

Repeat the same upon A 2.

Then repeat the process upon the other side of the diameter, commencing at B, and complete the figure.

FIG. 96.

To inscribe three equal Circles in an Equilateral Triangle, A B C, *each circle touching two of its sides.*

Bisect line A C in D, and draw line D B.

Bisect angle D C B and angle C D B by lines cutting each other in *c*.

Draw lines *c d*, *c e*, and *d e*, parallel to lines A C, A B, and B C, making an inscribed triangle, *c d e*.

From points *c d* and *e*, with the half of line *c d* as radius, describe circles as required.

FIG. 97.

To inscribe three equal Circles in an Equilateral Triangle, A B C, *each circle touching only one of its sides.*

Bisect the three sides, and draw lines B 1, C 2, and A 3; they will cut in D.

Bisect angle D A 1, and produce the bisecting line to cut line D 1 in *a*.

From D as centre, with D *a* as radius, cut lines 2 and 3 in *b* and *c*; *a*, *b*, and *c* will be the centres, and *a* 1 the radius of the three required circles.

By drawing through *a* a line parallel to line A C, cutting lines A 3 and C 2 in *d* and *e*, and from *d* and *e* drawing other lines parallel to lines A B and B C, the centres of three other circles equal to the first three may be obtained, and thus six circles inscribed in any equilateral triangle.

Fig. 98.

To inscribe four equal Circles in a Square, A B C D, *each circle touching two of its sides.*

Draw the diagonals A D and B C, intersecting at *c*.

Draw diameters *a b* and *d e* at *c*.

Draw the diagonals *a d*, *d b*, *b e*, and *a e*, of the four small squares into which the figure is divided.

Their points of intersections with the first diagonals A D, and B C, will be the centres, and the line *f g* the radius of the four required circles.

Fig. 99.

To inscribe three equal Circles in a Circle.

Inscribe an equilateral triangle, A B C.

Bisect the angles of the triangle by lines cutting its sides in *a*, *b*, *c*.

Join *a b* and *b c*, cutting A *c* and C *a* in D and E.

Draw line D E, and produce it cutting A B in F.

From A as centre, with radius A F, describe arc F G.

Bisect angle G A F, cutting arc F G in *d*.

Draw line F *d*, producing it to cut line A D in *e*.

e A is the radius of the required circles.

Set off from B and C, on the lines B *b* and C *a*, the radius *e* A, and describe the required circles.

Fig. 100.

To inscribe four equal Circles in a given Circle.

Draw the circumscribing square, A B C D.

Draw its diagonals, A D and B C.

Draw two diameters, E F and G H, parallel to its sides; all cutting in *a*.

The figure being now divided into four equal isoskeles triangles, bisect any angle of one, as *a* B G.

Produce the bisecting line, cutting line H G in *b*.

From *a* as centre, with *a b* as radius, cut the diameters in *c*, *d*, and *e*.

b, *c*, *d*, and *e* will be the centres, and line *b* G the radius of the four required circles.

Note.—By this figure four circles may be inscribed in a square, each circle touching only *one* of its sides.

Fig. 101.

To inscribe five equal Circles in a given Circle.

Divide the circumference, as for the inscription of a pentagon, into five equal parts.

From the centre C draw lines through two divisions, as 3 and 4, and produce them.

Bisect angle 3 C 4, and draw line A C, touching the periphery of the given circle in A.

At A, draw a tangent to the circle, cutting lines 3 and 4, and completing the triangle B C D.

Inscribe a circle in this triangle by Fig. 77, having its centre at *c*.

From C, with C *c* as radius, inscribe a circle D, cutting lines B C and C D in *a* and *b*.

From *c*, with the line *a b* as radius, cut the periphery of circle D in points *d*, 7, 8, D.

Upon those points, with radius *c* A, describe the circles to complete the figure.

Fig. 102.

To describe about a Circle six other Circles, each equal to the given circle, touching each other and the given circle.

From the centre of the given circle C, with its *diameter* as radius, describe a circle A.

With its radius cut its circumference in 1, 2, 3, 4, 5, and 6.

Upon those points, with the radius of the given circle, describe the six required circles.

Note.—By this figure six or seven equal circles may be inscribed in a given circle.

Let the outer circle, B D, be the given circle.

Draw any radius, C D.

Divide line C D into three equal parts, *a*, *b*, D.

From C as centre, with C *a* as radius, describe a circle.

From C, with C *b* as radius, describe a circle, and with its radius cut its circumference in 1, 2, 3, 4, 5, and 6.

Upon each point, with *a b* as radius, describe the six required circles.

Fig. 103.

To inscribe within a given Equilateral Triangle, D E F, *three equal Semicircles, having their diameters adjacent.*

Bisect the sides by lines A D, B E, and C F.

Inscribe the triangle, A B C, within the given triangle.

Upon A B describe a semicircle, *a b,* touching the sides of the triangle F E and F D.

From *a* and *b,* draw lines parallel to the sides of the triangle F D and F E, intersecting lines A D and B E in 1 and 2.

Draw line 1, 2, and construct upon it an equilateral triangle; its sides will be the adjacent diameters of the required semicircles.

Fig. 104.

To inscribe within a given Circle three equal Semicircles, having their diameters adjacent.

Draw a diameter, A B.

Draw a radius, C D, perpendicular to A B.

Trisect angle A C D in F and E.

Make D G equal D F.

Draw diameters from F and G.

Draw line E G, cutting line F in H.

Draw H I perpendicular to line A B.

Construct upon H I an equilateral triangle, its sides will be the adjacent diameters of the required semicircles.

FIG. 105.

To inscribe within a given Square four equal Semicircles, having their diameters adjacent and their arcs touching two sides of the given square.

Draw the diagonals, B and C.

Draw two central diameters perpendicular to each other.

Bisect A B in E.

Make C F equal A E.

Draw line F E, cutting diameter A in G.

Set off A G from the extremity of each diameter, as at *f f f*.

Join G*f f f*, and the lines so drawn will be the adjacent diameters of the required semicircles.

Note.—Four semicircles, their arcs touching *one* side of the given square, may be inscribed by Fig. 98, making *g* the centre and *g f* the radius.

ELEMENTARY AXIOMS AND FIGURES OF SECTION III.

Fig. 106.

The measurement of plane surfaces, or the superficial area of figures, is obtained by multiplying together two of their dimensions. If a figure has more than two different dimensions, as a Trapezium, then such figure is divided into triangles, the areas of such triangles found, and by their addition the sum of the area of the whole figure obtained. Suppose Fig. 106 to represent an oblong having a base, A B, four inches in length, and an altitude, A C, two inches in height; four inches and two inches are the dimensions of the two lines which bound its area or surface: therefore, multiply four inches by two inches, the quotient, eight square inches, will be the surface measurement of the oblong.

But this multiplication of the two *sides*, A B and A C, together, can apply only to figures whose angles are right angles, and the side, therefore, equal to the altitude, and whose opposite sides are equal.

FIG. 107.

All Parallelograms, whether right or oblique-angled, having equal bases and equal altitudes, have equal areas.

A B C D, Fig. 107, is a right-angled parallelogram, divided into four equal squares.

B C D E is an oblique-angled parallelogram.

Its base, C D, equal to the base of the right-angled parallelogram A B C D.

Its altitude, D B, equal to the altitude of the right-angled parallelogram.

By superposition, C D B is common to both parallelograms;

C A B is equal to D B E; therefore parallelogram A B C D is equal to parallelogram B C D E:

Therefore, the superficial areas of all parallelograms are obtained by multiplying together their base and altitude.

All parallelograms having equal altitudes, have to each other the same ratios that their bases have.

Parallelogram, A C 1, 1, Fig. 107, has the same altitude as parallelogram A B C D.

Its base is one half of the base of the parallelogram A B C D.

The area of A B C D is four, the area of A C 1, 1, is two.

Note.—The multiplication of *half* the *base* by the *whole altitude,* and the multiplication of the *whole base* *by half the altitude,* produce the same results. *Therefore, all parallelograms having equal bases have to each other the some ratios that their altitudes have.*

Fig. 108.

All Triangles are equal to Parallelograms of equal altitudes and half the base, or of equal base and half the altitude.

A B C, Fig. 108, is a triangle. A B D E a parallelogram having a base, A B, equal to the base of the triangle; an altitude, C F, equal to the altitude of the triangle.

By superposition A D C is equal to A F C;

B E C equal to B F C;

Therefore, in the parallelogram A B D E there are the triangles A B C, A D C, and B E C.

But A D C and B E C are together equal to A B C.

The area of the triangle A B C will, therefore, be half the area of the parallelogram A B D E.

Triangle A B G has the same base and altitude as triangle A B C.

The parallelogram A B E G is composed of triangle A E G and triangle A B G: each, by superposition, is equal to the other;

The area of the triangle A B G will be half the area of parallelogram A B E G;

But parallelogram A B E G, having the same base and altitude as parallelogram A B D E, is equal to it:

Therefore, triangle A B G is equal to triangle A B C; each being equal to the half of parallelogram A B D E.

Therefore triangles having equal bases and equal altitudes have equal areas.

The superficial areas of all triangles are obtained by multiplying together their base and *half* their altitude —or their altitude and *half* their base.

Fig. 109.

All Triangles having equal altitudes, have to each other the same ratios that their bases have; or, having equal bases, the same ratios that their altitudes have.

Triangle A B C, Fig. 109, has the same altitude as triangle A C D.

The base A B is half the base A D.

By superposition triangle A B C is half the triangle A D C.

The altitude of A B C is equal to the altitude of A D C.

The base of A B C is half the base of A D C.

Therefore the area of A B C is half the area of A D C.

Triangle A E D has the same base as triangle A C D.

The altitude B E is half the altitude B C.

The triangle A D E is common to triangle A D C.

By superposition triangle A F E is equal to triangle A H E.

Triangle F C E is equal to triangle H E B;

Triangle D E G equal to D E I;

Triangle G E C equal to B I E:

Therefore, the area of triangle A E D is *half* the area of triangle A D C.

Their bases being equal, and their altitudes in the ratio of the half to the whole.

Within a very minute degree of *precise accuracy*,* the circumference of a circle is equal to three times and one-seventh of its diameter. If the diameter be three inches and a half, its circumference will be eleven inches.

FIG. 110.

The Area of a Circle is equal to a Parallelogram, two of the sides of which are equal to half the circumference, and the other two to the radius of the circle, or the whole circumference and half the radius.

Circle A, Fig. 110, is divided into thirty-two equal parts or triangles, the radii of the circle forming their sides.

Sixteen of those parts or angles are opened out

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* The error is about $\frac{1}{2500}$ part of the circumference.

at B, making line B C equal to the half of the circumference of the circle A. The remaining sixteen, from 17 to 32 inclusive, will fit into the triangles on the line B C; the bases of triangles 17 to 32 being coincident with the vertices of triangles 1 to 16, and making a parallelogram equal in area to the circle.

The parallelogram B C, 17, 32, would be equal to a triangle having line B C for its base, and having twice the altitude of the parallelogram, which is the radius of the circle for its altitude.

Therefore a circle is equal to a triangle having half its circumference for base and its DIAMETER *for altitude;*

Or its whole circumference for base and its radius for altitude.

Or to a parallelogram having HALF *its circumference for base and its radius for altitude;*

Its whole circumference for base and HALF *its radius for altitude;*

Or its diameter for base and ONE-FOURTH *of its circumference for altitude.*

Fig. 111.

A Triangle having its base and altitude equal to the side of a square, and a Circle inscribed in that square, are to each other in the ratios of two, three, and four; that is, the Triangle will have one-half, and the Circle three-fourths of the area of the square.

The circumferences of circles have to each other the same ratios that their diameters have. Thus, if the diameter of a circle be 3½ inches, its circumference will be 11 inches.

If the diameter of another be 7, (twice 3½), its circumference will be 22, (twice 11); but the area of the first circle will be $9^{\text{in.}}\ 2'\ 3''$; the area of the second $36^{\text{in.}}\ 9'\ 0''$, thirty-six inches and nine-twelfths of an inch, *four* times the area of the first circle.

The diameter of the second circle is *twice* the diameter of the first; the area of the second *four* times the area of the first; and as four is the square of two, the areas of circles have the same ratios to each other as the squares of their diameters.

Thus, if the diameters of a series of circles are as 1, 2, 3, 4, 5, 6, &c., the *circumferences* of the second, third, fourth, fifth, and sixth will be respectively twice, three, four, five, and six times the *circumference* of *the first.*

The areas of the circles being—
Of the second, four;
Of the third, nine;
Of the fourth, sixteen;
Of the fifth, twenty-five;
And of the sixth, thirty-six times the area of the first. Those numbers being the squares of 2, 3, 4, 5, and 6.

Fig. 112.

To draw a Right Line equal to half the circumference of a given Circle.

Draw a diameter, A B.

Draw line A C perpendicular to line A B, and equal to three times the radius of the circle.

Draw a line at B perpendicular to A B.

With the radius of the circle cut off arc B D.

Bisect arc B D.

Draw a line from the centre of the circle through the bisection, cutting line B in E.

Join E C.

Line E C will be equal to half the circumference of the given circle.

Fig. 113.

Two Right Lines, A B *and* C D, *being given, to find a mean proportional line.*

Produce line A B, making B E equal line C D.

Bisect line A E in F.

From F, with radius F A, describe a semicircle on line A E.

At B, the point of junction of the two given lines, erect a line perpendicular to A E, cutting the semicircle in G.

Line B G will be a mean proportional to lines A B and C D.

Line B G being in the same proportion greater than line C D, that it is less than line A B.

If line A B were four inches in length, and line C D one inch, then B G would be two inches in length.

FIG. 114.

Two right lines, A B *and* C D, *being given, to find a third line proportional to them, when the required line is* LESS *than either of the given lines.*

Make line E F equal to the given line A B.

Draw line E G, making any angle with line E F, equal to line C D.

Draw line F G.

From E, with radius E G, cut line E F in H.

Draw line H parallel to line F G, cutting line E G in I.

Line E I will be the third proportional line required, less than either of the given lines.

If lines E I *and* E G *are the given lines, and a third line proportional to them and greater than either is required,*

Draw line E G.

Cut off upon it line E I.

From E, with radius E G, describe an arc, G H.

From I draw a line cutting the arc in any point, as H.

Draw a line from E through H, and produce it.

Draw a line from G parallel to line I H, cutting the line drawn from E through H in F.

Line E F will be the third proportional line required, greater than either of the given lines.

Note.—*This is, in fact, merely a reversal of the process.*

Fig. 115.

Three Right Lines, A B, C D, *and* E F, *being given, to find a fourth line proportional to them.*

Make line G H equal the given line A B. Draw line G I, equal to line C D, making any angle with line G H.

Draw line H I.

From G cut off line E F on line G H, making G K equal line E F.

From K draw a line parallel to line H I, cutting line G I in L.

Line G L will be the fourth proportional line required.

If G I, G K, *and* G L *are the given lines, and a fourth proportional to them greater than either is required,*

From G draw a line making any angle with line G I.

Cut off upon it line G K.

Upon G I cut off G L.

Draw line K L.

From I draw a line parallel to line K L, cutting the produced line G K in H.

Line G H will be the fourth proportional, greater than the given lines.

Fig. 116.

To divide a right line, A B, *into extreme and mean proportion.*

Erect a perpendicular at A, making line A C equal *half* A B.

Draw line B C.

From C, with radius C A, describe an arc, cutting line B C in D.

From B, with radius B D, describe an arc, cutting line A B in E.

Line E A is the extreme, and line B E the mean, proportional of line A B.

That is, A E has the same ratio to B E, that B E has to the whole line A B.

Fig. 117.

To divide a right line A B *successively into its* $\frac{1}{2}$, *its* $\frac{1}{3}$, *its* $\frac{1}{4}$, *its* $\frac{1}{5}$, *&c. &c.*

Upon A B construct any parallelogram, either right or oblique-angled, as A B C D.

Draw diagonals C B and A D, intersecting in 2.

Draw line 2 parallel to A C, cutting A B in $\frac{1}{2}$.

Draw line D $\frac{1}{2}$, cutting B C in 3; draw line 3 parallel to A C, cutting A B in $\frac{1}{3}$.

Draw line D $\frac{1}{3}$, cutting line B C in 4; draw line 4 parallel to A C, cutting A B in $\frac{1}{4}$, &c. &c.

The divisions thus obtained are the $\frac{1}{2}$, the $\frac{1}{3}$, the $\frac{1}{4}$, the $\frac{1}{5}$, &c., of the whole line A B.

It has been shown that,—*All Parallelograms having equal bases and equal altitudes have equal areas:*

Therefore, any square may be converted into a rhombus, or any oblong into a rhomboid of equal area, by the base of the square or oblong being made the base of the rhombus or rhomboid, and the altitude of the square or oblong the altitude of the rhombus or rhomboid.

That,—*All Triangles having equal bases and equal altitudes have equal areas:*

Therefore, triangles may be converted into other triangles (as an equilateral into a scalene, or a scalene into an isoskeles), of equal area, by the base and altitude of the given triangle being made the base and altitude of the required triangle.

That,—*All Triangles are equal to Parallelograms of equal altitude and half the base, &c.:*

Therefore, any triangle may be converted into a parallelogram of equal area, the base of the triangle being made the base of the parallelogram, and *half* the altitude of the triangle being made the altitude of the parallelogram, &c. Or any parallelogram may be converted into a triangle of equal area; the base of the parallelogram being made the base of the triangle, and twice the altitude of the parallelogram the altitude of the triangle, &c

That,—*Circles have areas equal to Parallelograms formed by two sides, each equal to half their circumferences, and the remaining two, each equal to their radii, or equal to Triangles formed by half the circumference as base, and the diameter as altitude:*

Therefore, any circle may be converted into a parallelogram or triangle of equal area.

CONSTRUCTIVE PROBLEMS OF SECTION III.

Fig. 118.

To construct a Triangle equal to a given Trapezium, A B C D.

Produce the base of the given trapezium, B A, in the direction of E.

Draw a diagonal, A D.

From C (the angle opposite the diagonal), draw a line parallel to the diagonal, and cutting the produced base in E.

Draw line E D.

The triangle E D B is equal in area to the given trapezium.

Therefore, a trapezium may be converted into a parallelogram, by making *half* the base of the obtained triangle and its altitude the base and altitude of the parallelogram.

Fig. 119.

The same method may be applied to change any given Rectilineal Figure, regular or irregular, into another figure, equal in area, but one side less.

A B C D E F is a regular hexagon.

Draw a line A E from the alternate angles A and E.

Produce the base A B, in both directions.

Draw a line from F parallel to line A E, cutting the produced base in G.

Draw line G E.

G B C D E *is an irregular pentagon, equal in area to the regular hexagon,* A B C D E F.

Draw line B D.

Draw a line from C, parallel to B D, cutting the produced base in H. Join D H and a trapezoid, G E D H, is obtained, equal in area to the regular hexagon and the irregular pentagon.

Draw line E H from the alternate angles E and H.

Produce the base G H.

Draw from D a line parallel to line E H, cutting the produced base in I.

Draw line E I.

And G E I is a triangle equal in area to the hexagon, pentagon, and trapezoid.

By this method any polygon may be transformed into a triangle having equal area.

Therefore, any polygon may be transformed into a parallelogram, by making *half* the base and the altitude of the obtained triangle the base and altitude of the parallelogram.

Fig. 120.

To construct a Triangle equal in area to any given regular Polygon, A B C D E F.

Construct the elementary triangle of the polygon A B F; that is, from its centre draw radii to the extremities of one side, as A and B.

Draw a right line, A B C D E A, equal to the whole perimeter of the polygon.

Construct upon it a triangle A E F, equal to the elementary triangle of the polygon.

Five of such triangles constructed upon the line A B C D E A, would be the exact representatives of the triangles contained in the polygon.

But it has been shown in Fig. 105, that—*All triangles, having equal altitudes, have to each other the same ratio as their bases have.*

Therefore, draw line F A, making triangle A F A equal to the whole five triangles, of which this polygon is composed.

Therefore, triangle A F A is equal in area to the given polygon, A B C D E.

FIG. 121.

On a given base A B, *or with a given altitude, to construct a Triangle equal in area to a given Triangle,* C D E.

Draw line D F, the altitude of the given triangle.

Find, by Fig. 115, a fourth line proportional to
Line A B, the *given* base of the *required* triangle;
Line C E, the *base* of the *given* triangle; and
Line D F, the *altitude* of the *given* triangle.

This fourth proportional line will be the altitude of the required triangle.

As in Fig. 115, draw line B G equal to line C E, at any angle to A B.

Draw line A G.

Cut off B H, equal to line D F.

Draw line H parallel to A G, cutting B G in I.

Line B I will be the altitude, and A B the base of the required triangle, equal in area to the given triangle.

That is, Triangle A F B is equal in area to Triangle C D E.

If the *altitude* of the required triangle be given, find a fourth proportional to the base of the given triangle, its altitude, and the given altitude of the required triangle: such fourth proportional will be the *base* of the required triangle.

But a triangle is equal to a parallelogram of equal altitude and half the base. Therefore, a triangle may be constructed on a *given line,* equal to a given parallelogram, by making first a triangle equal to the given parallelogram, and then constructing on the given line a triangle equal to the first triangle by the last figure. A triangle may be constructed equal in area to a trapezium, by Fig. 118. Therefore, a triangle may be constructed on a *given line* equal to a given trapezium. A triangle may be constructed equal in area to any polygon, by Fig. 114 or 115.

Therefore, a triangle may be constructed on a *given line* equal to any given polygon.

A circle is equal in area to a triangle, &c. Therefore, a triangle may be constructed on a *given line* equal to a given circle.

FIG. 122.

To construct an Equilateral Triangle equal to a given Triangle, A B C, *not Equilateral.*

Upon the side of the given triangle, A B, construct an equilateral triangle A B D.

Produce line D B.

Draw a line from C (the vertex of the given triangle) parallel to the side A B, and cutting the produced line D B in E.

By Fig. 113 find a mean proportional line to lines D B and D E. That is, upon D E describe a semicircle.

At B, draw B F perpendicular to D E.

Line B F will be the mean proportional to lines D B and B E, and will be the side of an equilateral triangle, equal in area to the given triangle, as triangle B G H.

Therefore any parallelogram, trapezium, polygon or circle, may be converted into an *Equilateral* triangle having the, same area.

Fig. 123.

On a given line A B *to construct a Parallelogram equal to a given Parallelogram,* C D E F.

Find by Fig. 115, or as in Fig. 121, a fourth proportional to the given line A B;

The base, C D, of the given parallelogram;

And the altitude, C E, of the given parallelogram:

Such fourth proportional, as B G, will be the altitude of the required parallelogram.

But a parallelogram is equal to a triangle of equal base, and twice the altitude of the parallelogram.

Therefore, a parallelogram may be constructed on a given line equal in area to a given triangle, by making a parallelogram equal to the given triangle, and then constructing a parallelogram on the given line equal to the obtained parallelogram.

Therefore, a parallelogram may be constructed on a given line equal in area to a square, a trapezium, a polygon, or a circle, by resolving those figures into their equivalent triangles, the obtained triangles into equivalent parallelograms, and then constructing on the *given lines* parallelograms equal to the obtained parallelograms.

Fig. 124.

Another method to construct a Parallelogram on a given line A B, *equal in area to a given Parallelogram,* C D E F.

Produce the base of the given parallelogram C D to G, making C G equal to A B.

Draw a line from G, cutting angle E of the given parallelogram, and produce it.

Produce side D F of the given parallelogram, cutting the diagonal from G in H.

Line F H will be the second side of the required parallelogram; therefore the parallelogram, C I G K, constructed of the given line A B and the obtained line F H, will be equal in area to the given parallelogram C D E F.

Fig. 125.

To construct a Parallelogram upon a given line B E, *proportionally to another given Parallelogram,* A B C D.

Draw the diagonal B C.

At E draw a line E F parallel to the side of the given parallelogram B D, cutting the diagonal in F.

From F draw a line F G parallel to the base of the parallelogram A B.

B E F G will be a parallelogram constructed on the given line B E, and proportional to the given parallelogram. That is, the side E F will bear the same proportion to the side B D that the base B E bears to the base A B.

Fig. 126.

To construct a Square equal in area to a given Parallelogram, A B C D.

Find by Fig. 113, or as in Fig. 122, a mean proportional to the base A B and the altitude A C of the given parallelogram.

Such mean proportional, as A E, will be the side of the required square.

But a parallelogram is equal to a triangle of the same base and twice the altitude.

Therefore, a given triangle may be converted into a square having equal area, by making a parallelogram equal to the given triangle, and constructing a square equal to the obtained parallelogram.

And all other figures, as trapezia, polygons, and circles, may be, by the same method, converted into squares.

Fig. 127.

To construct a Parallelogram on a given line A B, *having equal area to a given Square,* F.

Find, by Fig. 114, a third proportional to the given line A B and the side of the square F. Such third proportional will be the altitude of the required parallelogram.

That is, as in Fig. 114,

Draw line B C at any angle to line A B.

Make line B C equal to one side of the square.

Draw line A C joining the extremities of the two lines.

From B, with B C as radius, describe arc C D, making B D equal to B C.

Draw from D a line parallel to line A C, cutting B C in E.

Line B E will be the required side of the parallelogram.

That is, a parallelogram having line A B as base, and line B E as altitude, will be equal to the given square F.

Fig. 128.

To draw a Parallelogram equal to a given Triangle, A B C, *in area and perimeter.*

From the vertex, B, draw a line, B D, perpendicular to the base, A C.

B D is the altitude of the triangle.

Produce side A B, making B E equal to B C.

The line A E is equal to the two sides A B and B C of the triangle.

Bisect the base A C, as at F.

Bisect the line A E, as at G.

Draw a line, as H, equal to half A C.

Draw another line, as I, parallel to H, and at a distance from it, equal to line B D, the altitude of the triangle.

From both extremities of line H, with the half of A E as radius, cut line I in 1 and 2.

Draw lines from 1 and 2 to the adjacent extremities of line H, and a parallelogram will be obtained equal to the given triangle in area and perimeter.

Fig. 129.

To construct a Triangle, Parallelogram, Circle, or regular Polygon, having any given proportion in area greater or less than a given Triangle, Parallelogram, Circle, or regular Polygon.

Let the line A B represent the base or side of a triangle or parallelogram, or the diameter of a circle or polygon.

Let the figure first proposed to be enlarged or diminished be a square, as D.

Let the required figure be proposed to be one-third of the given figure.

Produce the line A B indefinitely.

Take $\frac{1}{3}$ of A B, and set it off from B as at $\frac{1}{3}$.

Bisect A $\frac{1}{3}$ as at C.

From C, as centre, with C A as radius, describe a semicircle.

At B erect a perpendicular, cutting the semicircle at x.

B x will be the side of a square, having $\frac{1}{3}$ the area of the given square, of which A B is the side, as square *d*.

If the proposed figure be a triangle, as E, A B being its base,

With the obtained line, B x, as base, construct a triangle having angles equal to those of the given triangle, as triangle *e*.

If the proposed figure be an oblong, as F, A B being its base,

With the obtained line, B x, construct an oblong proportional to the given oblong, as *f*; or, having obtained a proportional base, B x, obtain a proportional side in a similar manner.

If the proposed figure be a circle, as G, or a polygon, of which A B is the diameter,

On the obtained line, B x, as *diameter*, describe a circle, as *g*.

It is proposed to construct or describe a figure, having $1\frac{1}{2}$ the area of the given figure represented by A B.

From B set off $1\frac{1}{2}$ A B, as at 1 and $\frac{1}{2}$.

Bisect A $\frac{1}{2}$, and at the point of bisection, with half A $\frac{1}{2}$, describe a semicircle.

Erect a perpendicular at B, cutting the semicircle, as at *y*.

B *y* will be the side required, on which to construct, as given in detail with the first figure, a figure having $1\frac{1}{2}$ the area of a similar figure constructed on line A B.

It is proposed to construct or describe a figure having twice the area of a given similar figure constructed on A B.

From B set off twice A B, as at 2.

Bisect A 2, and describe, with the half of A 2, a semicircle as before.

Erect the perpendicular again at B, produce it to cut the semicircle as at *z*.

B *z* will be side required on which to construct, &c. &c., as before.

It is proposed to construct a figure having $2\frac{2}{5}$ the area of a given similar figure constructed on A B.

Set off from B twice A B, as at 2, and $\frac{2}{5}$ of A B, as at $\frac{2}{5}$.

Bisect A $\frac{2}{5}$ and proceed as before.

Therefore the proportion desired has *always to be added to* B, as $\frac{1}{3}$—$1\frac{1}{2}$—2—$2\frac{2}{5}$.

In this figure it is essential to remember that,—

If a semicircle were described from B, the perpendicular erected at B, and touching the semicircle, would be equal to A B. Therefore, that

All semicircles described upon diameters less than A 1 give, at B, perpendiculars less than A B.

B $\frac{1}{2}$ is the side of a figure which would commonly be said to be *half as large again* as a similar one on A B. But B *y* is the side of a figure *really* half as large again.

B 2 would also be commonly said to give a figure *twice* as large as a similar figure on A B. But B *z* is the *true side* of a figure, having twice the area of the first figure.

The centre of the semicircle, therefore, varies with the varying proportion required; *but the point at which to erect the perpendicular never changes, it is always at the extremity of* A B.

Fig. 130.

To construct a Triangle equal in area to the areas of two or more dissimilar Triangles, as Triangles A B C *and* D.

By Fig. 121, change triangle D into an equal triangle, having its altitude equal to the altitude of triangle A B C, as E.

Produce the base B A of triangle A B C, making B F equal to the base of triangle E.

Join C F.

Triangle A C F is equal in area to the two given triangles.

The number of the triangles may, of course, be increased at pleasure.

To construct a Triangle equal in area to the difference in the area of two given unequal Triangles, B C F *and* D.

REFER TO THE LAST FIGURE.

Proceed, as in the last figure, to convert triangle D into an equal triangle, having its altitude equal to that of triangle B C F.

From F set off, within the triangle, the base of the obtained triangle, as at A.

Triangle A B C is equal to the difference in the areas of triangles B C F and D.

Note.—Fig. 130 is the addition of the base of the obtained triangle to that of the given triangle.

Fig. 131 is the subtraction of the base of the obtained from that of the given triangle.

FIG. 131.

To construct a Triangle, Parallelogram, Circle, or regular Polygon, equal to two given similar Triangles, Parallelograms, Circles, or regular Polygons.

Let A B and C D represent the bases of two similar

triangles, the sides of two squares, or the diameter of two circles or regular polygons.

With lines A B and C D construct a right angle, as A B E.

Join B E.

The line B E will be the base of a triangle, the side of a square, or the diameter of a circle, or regular polygon, equal in area to the two triangles, squares, circles, or polygons, constructed, or described, upon the two lines, A B and C D.

Fig. 132.

To construct a Triangle, Parallelogram, Circle, or regular Polygon, equal in area to the difference between two given similar Triangles, Parallelograms, Circles, or regular Polygons.

Let A B and C D represent the bases of two similar triangles, the sides of two squares, or the diameters of two circles or regular polygons.

At B erect a perpendicular to A B.

From A, with C D as radius, describe an arc, cutting the perpendicular at B in E.

The line B E will be the base of a similar triangle, the side of a square, or the diameter of a circle or regular polygon, equal in area to the difference between the areas of the triangles, squares, circles, or polygons constructed, or described upon the two lines A B and C D.

FIG. 133.

To change any Rectilineal Figure, regular or irregular, into another equal Figure, having one side more.

To commence with the triangle A B C,

Assume a point, as D, for one of the angles of the figure (four-sided) to be obtained.

Draw B D.

Draw a line from B parallel to, and lying in the same direction as, D C.

Draw a line from C parallel to line B D, and cutting line B in E.

Join D E.

Figure A B E D will be equal to figure A B C.

Assume a point F for one of the angles of the figure (five-sided) to be obtained.

Draw B F.

Draw a line from A parallel to B F.

Draw E F.

From B, draw a line parallel to line E F, cutting the line from A in G.

Join G F.

Figure G B E D F will be equal to figure A B E D.

Assume a point H for one of the angles of the figure (six-sided) to be obtained.

Draw E H.

Draw a line from D parallel to line E H.

Draw B H.

Draw a line from E parallel to line B H, cutting the line from D in I.

Join I H.

Figure G B E I H F, is equal to all the figures, five, four, and three-sided.

Note.—The new angular point has in this use of the figure always to be assumed.

Fig. 134.

To construct a Figure equal in area to the sum of the areas of any two dissimilar Figures.

This result may be obtained by Fig. 119, by reducing each figure into an equal triangle, and then by constructing, by Fig. 130, a triangle equal to the two obtained triangles.

Or, by an application of Fig. 119,

Let A B C D E, and 1, 2, 3, be two irregular figures, to be converted into one equal figure.

Join A 2.

Draw from 1 a line parallel to A 2, to cut line A B in 4.

Join 2, 4.

Join 2 E.

Draw from 3 a line parallel to line 2 E, to cut line E D in 5.

Join 5, 2.

B C D, 5, 2, 4, will be *one* figure, equal in area to the *two* given figures.

Join D B.

Draw a line from C parallel to D B, cutting line 4 B in 6.

Join D 6, and the *two* irregular figures have been changed into a pentagon; by a repetition of the process change this into a four-sided figure, and that into a triangle.

The triangle may be converted into an equal parallelogram, and that into an equal square.

Thus any number of irregular rectilineal figures may be converted into an equal square.

By reducing any irregular rectilineal figures into equal triangles, and then constructing, by Fig. 130, a triangle equal to the difference between the two obtained triangles, a figure may be obtained equal to the difference between any two or more irregular figures.

By a reversal of the process, any rectilineal figure, regular or irregular, may be converted into an equal figure, having one side more than the given figure.

Fig. 135.

To divide a given Triangle, A B C, *into any given number of equal or proportional parts, by lines drawn from a given point* D, *in one of its sides.*

Draw a line from D to the opposite angle C.

Divide the side of the triangle on which point D is situate, as line A B, into the same number of similar parts, whether equal or proportional, as those into which the triangle is proposed to be divided, as 1, 2, 3.

Draw lines from 1, 2, 3, parallel to line D C, cutting the sides of the triangle in 4, 5, 6.

Draw lines from D to points 4, 5, 6.

Rub out lines 1 4, 2 5, D C, 3 6, and the figure will be completed.

That is, D A 4, D 4 5, D 5 6, D 6 B, will bear the same proportion to the given triangle A B C, that the parts A 1, 1 2, 2 3, and 3 B, bear to the side, A B, of the given triangle.

If it be required to divide a given triangle into a number of parts in similar proportion to a given divided line, greater or less than the side of the triangle, as line E F; then, by Fig. 15, divide line A B similarly to line E F, and proceed.

FIG. 136.

To divide a given Rectilineal Figure, as A B C D E, *into any given number of equal or proportional parts, by lines drawn from a given angle of it, as* B.

Reduce the given figure into an equal triangle, by Fig. 119, making the given point, B, its vertex.

Divide the base of the obtained triangle, A 4, into the same number of similar parts, whether equal or proportional, as those into which the given figure is proposed to be divided, as 1, 2, 3, 4.

Draw lines B C and B D.

Produce side C D.

Draw lines 2 and 3, parallel to line B C, cutting line C D in 5 and 6.

Draw a line from 6 parallel to line B D, cutting line D E in 7.

Draw lines B 1, B 5, and B 7.

Rub out lines B C and B D, and the figure will be completed.

Fig. 137.

To divide a given Rectilineal Figure, A B C D E F, *into any given number of equal or proportional parts, by lines drawn from a given point,* G, *within it.*

Reduce the given figure into an equal triangle, as in the last case, by Fig. 119.

Reduce the obtained triangle into an equal triangle, having its vertex in the given point G, by Fig. 121, as triangle *o* G 5.

Divide the base of the triangle *o* 5 into the same number of similar parts, whether equal or proportional, as those into which the given figure is proposed to be divided, as 1, 2, 3, 4, 5, as in the last figure.

Produce F C.

Draw line G F.

Draw lines from 4 and 5 parallel to line G F, and cutting the produced line F C in 6 and 7.

Draw line G C.

Draw a line from 7 parallel to line G C, and cutting line B C in 13.

Produce line E D.

Draw line G E.

Draw lines from 1 and 2 parallel to line G E, cutting the produced line E D in 8 and 9.

Produce line D A.

Draw line G D.

Draw lines from 8 and 9 parallel to line G D, and cutting line D A in 10 and 11.

Draw line G A.

Draw a line from 11 parallel to line G A, and cutting A B in 12.

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London: — Printed by G. Barclay, Castle St. Leicester Sq.